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On the metric dimension of a total graph of non-zero annihilating ideals

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Abstract

Let R be a commutative ring with identity which is not an integral domain. An ideal I of a ring R is called an annihilating ideal if there exists $r \in R - \{0\}$ such that $Ir = (0)$. Visweswaran and H. D. Patel associated a graph with the set of all non-zero annihilating ideals of R, denoted by $\Omega(R)$ as the graph with the vertex-set $A(R)^*$, the set of all non-zero annihilating ideals of R and two distinct vertices I, J are joined if and only if $I+J$ is also an annihilating ideal of R. In this paper, we study the metric dimension of $\Omega(R)$ and provide metric dimension formulas for $\Omega(R)$.

1 Introduction

Assigning a metric dimension to a graph was first introduced by Harary and Melter in [10]. Later, this concept was applied to graphs associated to commutative rings. (see, for example [7, 8, 9]). In this paper, we study the metric dimension of a total graph of non-zero annihilating ideals.

Throughout this paper, all rings are assumed to be commutative with identity and they are not fields. The sets of all zero-divisors, nilpotent elements, minimal prime ideals, maximal ideals and Jacobson radical of R are denoted by $Z(R)$, $\mathrm{Nil}(R)$, $\mathrm{Min}(R)$, $\mathrm{Max}(R)$ and $J(R)$, respectively. For a subset T of a ring R we let $T^* = T - \{0\}$. An ideal with non-zero annihilator is called an

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annihilating ideal. The set of annihilating ideals of R is denoted by $\mathbb{A}(R)$. For every subset I of R, we denote the *annihilator* of I by $ann(I)$. Some more definitions about commutative rings can be find in [4, 5].

We use the standard terminology of graphs following [15]. By $G = (V, E)$, we mean a graph, where V and E are the set of vertices and edges, respectively. If we can find at least one path between two any vertices of G, then G is called *connected*. Also, the length of the shortest path between two distinct vertices x and y is denoted by $d(x, y)$ (note that $d(x, y) = \infty$, if there is not any path between x and y) and $\mathrm{diam}(G) = max\{d(x, y) \,|\, x, y \in V\}$ is called the *diameter* of G.

The *girth* of a graph G, denoted by $\mathrm{girth}(G)$, is the length of the shortest cycle in G. The graph $H = (V_0, E_0)$ is a subgraph of G if $V_0 \subseteq V$ and $E_0 \subseteq E$. Moreover, H is called an *induced* subgraph by V_0, denoted by $G[V_0]$, if $V_0 \subseteq V$ and $E_0 = \{\{u, v\} \in E \,|\, u, v \in V_0\}$. Let $x \in V$, then $N(x) = \{y \in V \,|\, \{x, y\} \in E\}$ and $N[x] = N(x) \cup \{x\}$.

Let $G = (V, E)$ be a connected graph, $S = \{v_1, v_2, \ldots, v_k\}$ be an ordered subset of V and $v \in V(G) \setminus S$. The metric representation of v with respect to S is the k-vector $D(v|S) = (d(v, v_1), d(v, v_2), \ldots, d(v, v_k))$. For $S \subseteq V$, if for every $v, u \in V(G) - S$, $D(u|S) = D(v|S)$ implies that $u = v$, then S is called the resolving set for G. The metric basis for G is a resolving set S of minimum cardinality and the number of elements in S is called the metric dimension of G ($dim_M(G)$).

For a graph G with $|V(G)| \geq 2$, if for all $x \in V(G) - \{u, v\}$, $d(u, x) = d(v, x)$ (u, v are two distinct vertices), then u, v are distance similar. Clearly, if either $u - v \notin E(G)$ and $N(u) = N(v)$ or $u - v \in E(G)$ and $N[u] = N[v]$, then two distinct vertices u and v are distance similar.

An *k-partite* graph is one whose vertex set can be partitioned into k subsets so that an edge has both ends in no subset. A *complete k-partite* graph is an k-partite graph in which each vertex is adjacent to every vertex that is not in the same subset. The complete bipartite (i.e., 2-partite) graph with part sizes m and n is denoted by $K^{m,n}$. If $m = 1$, then the bipartite graph is called star graph. A *complete* graph is a graph such that there exist an edge detween each pair of vertices and is denoted by K^n.

Let R be a commutative ring with identity which is not an integral domain. An ideal I of a ring R is called an annihilating ideal if there exists $r \in R - \{0\}$ such that $Ir = (0)$. Visweswaran and H. D. Patel associated a graph with the set of all non-zero annihilating ideals of R, denoted by $\Omega(R)$ as the graph with the vertex-set $A(R)^*$, the set of all non-zero annihilating ideals of R and two distinct vertices I, J are joined if and only if $I + J$ is also an annihilating ideal of R. In this paper, we study the metric dimension of $\Omega(R)$ and provide metric dimension formulas for $\Omega(R)$.

2 Metric dimension of a total graph of a reduced ring

Let R be a commutative ring. In this section, we provide a metric dimension formula for a total graph of non-zero annihilating ideals when R is reduced.

Lemma 2.1. *Suppose that R is a commutative ring with identity and $\Omega(R)$ is connected. If R is not an integral domain, then the following statements are equivalent.*

(1) $\dim_M(\Omega(R))$ *is finite.*

(2) *R has only finitely many ideals.*

Proof. (1) $\Rightarrow$ (2) Assume that $dim_M(\Omega(R))$ is finite and for some non-negative integer n, let $W = \{I_1, I_2, \dots, I_n\}$ be the metric basis for $\Omega(R)$. Since $diam(\Omega(R)) \leq 2$ (see [14]), for every $I \in A(R)^* - W$, there are only $(2+1)^n$ choices for $D(I|W)$. So $|A(R)^*| \leq 3^n + n$ and hence R has only finitely many ideals.

(2) $\Rightarrow$ (1) is clear. □

If R is a reduced ring with finitely many ideals, then R is Artinian ring and so by [4, Theorem 8.7], R is direct product of finitely many fields. Using this, we prove the following result.

Theorem 2.1. *Suppose that R is a reduced ring with identity that is not an integral domain. If $\Omega(R)$ is connected and $dim_M(\Omega(R))$ is finite, then:*

(1) *If $|\mathrm{Max}(R)| = 3$, then $dim_M(\Omega(R)) = 2$.*

(2) *If $|\mathrm{Max}(R)| = n \geq 4$, then $dim_M(\Omega(R)) = n$.*

Proof. (1) If $n = 3$, then $R \cong F_1 \times F_2 \times F_3$, where F_i is a field for every $1 \leq i \leq 3$. Now, we put $W = \{(0) \times F_2 \times F_3, F_1 \times (0) \times F_3\}$ and by Figure 1, we can easily get

$D((0) \times F_2 \times (0)|W) = (1, 2)$,

$D(F_1 \times (0) \times (0)|W) = (2, 1)$,

$D((0) \times (0) \times F_3|W) = (1, 1)$,

$D(F_1 \times F_2 \times (0)|W) = (2, 2)$.

So for every $x, y \in V(\Omega(R)) - W$, $D(x|W) \neq D(y|W)$ and hence $dim_M(\Omega(R)) = 2$.

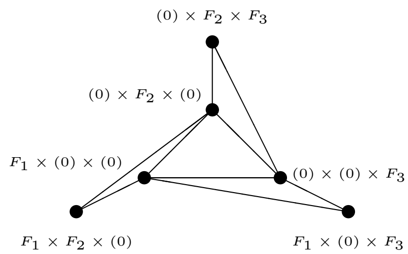


Figure 1: $\Omega(F_1 \times F_2 \times F_3)$

(2) Assume that $n \geq 4$. By Lemma 2.1, $R \cong F_1 \times \cdots \times F_n$, where F_i is a field for every $1 \leq i \leq n$. We show that $dim_M(\Omega(R)) = n$. Indeed, we have the following claims:

Claim 1. $dim_M(\Omega(R)) \geq n$.

Since R is direct product of finitely many fields, by Lemma 2.1, $dim_M(\Omega(R))$ is finite. So we can let for some integer k, $W = \{I_1, I_2, \ldots, I_k\}$ be the metric basis for $\Omega(R)$. On the other hand, since, $diam(\Omega(R)) \in \{1, 2\}$ (see [14]), for every $I \in A(R)^* - W$, there are only 2^k possibilities for $D(I|W)$. This implies that $|A(R)^*| - k \leq 2^k$. Since $|A(R)^*| = 2^n - 2$, $2^n - 2 - k \leq 2^k$ and hence $2^n \leq 2^k + 2 + k$. Since $n \geq 4$, we have $k \geq n$. Therefore, $dim_M(\Omega(R)) \geq n$.

Claim 2. $dim_M(\Omega(R)) \leq n$.

For every $1 \leq i \leq n$, let $(I_1, I_2, \ldots, I_n) = \mathfrak{m}_i \in A(R)^*$ such that $I_i = 0$ and $I_j = F_j$, for every $1 \leq j \leq n$ with $i \neq j$. We put $W = \{\mathfrak{m}_1, \mathfrak{m}_2, \ldots, \mathfrak{m}_n\}$ (in fact $W = Max(R)$). We show that W is the resolving set for $\Omega(R)$. For this, let $I, J \in V(\Omega(R)) - W$ and $I \neq J$. We need only to show that $D(I|W) \neq D(J|W)$. Let $I = (I_1, I_2, \ldots, I_n)$ and $J = (J_1, J_2, \ldots, J_n)$. Since $I \neq J$, for some $1 \leq i \leq n$, $I_i = 0$ and $J_i = F_i$ or $I_i = F_i$ and $J_i = 0$. Without loss of generality, we can assume that $I_1 = 0$ and $J_1 = F_1$. Now, it is easy to see that $d(I, \mathfrak{m}_1) = 1$ and $d(J, \mathfrak{m}_1) = 2$. This clearly shows that $D(I|W) \neq D(J|W)$. Therefore $dim_M(\Omega(R)) \leq n$.

Now, by Claim 1, 2 we have $dim_M(\Omega(R)) = n$, for $n \geq 4$. $\square$

3 Metric dimension of a total graph of a non-reduced ring

In this section, we study the metric dimension of $\Omega(R)$ when R is non-reduced. We begin with the following useful lemma.

Lemma 3.1. *Let $R \cong R_1 \times \cdots \times R_n$, where R_i is an Artinian local ring for every $1 \leq i \leq n$. and let $I = (I_1, \ldots, I_n)$ and $J = (J_1, \ldots, J_n)$. Then*

(1) *$I - J$ is an edge of $\Omega(R)$ if and only if for some $1 \leq i \leq n$, $I_i, J_i \subseteq \mathrm{Nil}(R_i)$.*

(2) *If $0 \neq I \subseteq J(R)$, then I is adjacent to all other vertices in $\Omega(R)$.*

Proof. (1) Let $I - J$ be an edge of $\Omega(R)$. If for every $1 \leq i \leq n$, $I_i \nsubseteq \mathrm{Nil}(R_i)$ or $J_i \nsubseteq \mathrm{Nil}(R_i)$, then for every $1 \leq i \leq n$, $I_i = R_i$ or $J_i = R_i$. This implies that $I + J = R$ and hence $I - J$ is not an edge of $\Omega(R)$, a contradiction. The converse is clear.

(2) By Part (1) is clear. □

Remark 3.1. *For a connected graph G, if $V_1, V_2, \ldots, V_k$ is a partition of $V(G)$ such that for every $1 \leq i \leq k$, $x, y \in V_i$ implies that $N(x) = N(y)$. Then $dim_M(G) \geq |V(G)| - k$.*

Theorem 3.1. *Suppose that $R \cong R_1 \times \cdots \times R_n$, where R_i is an Artinian local ring such that for every $1 \leq i \leq n$, $|A(R_i)^*| \geq 1$. Then $dim_M(\Omega(R)) = |A(R)^*| - 2^n + 1$.*

Proof. Suppose that $I = (I_1, \ldots, I_n)$ and $J = (J_1, \ldots, J_n)$ are vertices of $\Omega(R)$. Define the relation $\sim$ on $V(\Omega(R))$ as follows: $I \sim J$, whenever for each $1 \leq i \leq n$, "$I_i \subseteq Nil(R_i)$ if and only if $J_i \subseteq Nil(R_i)$".

Clearly, $\sim$ is an equivalence relation on $V(\Omega(R))$. The equivalence class of I is denoted by $[I]$. Suppose that X and Y are two elements of the equivalence class of I. Since $X \sim Y$, by part (1) of Lemma 3.1, we can easily get $N(X) = N(Y)$. Now, since the number of equivalence classes is $2^n - 1$, then

$$dim_M(\Omega(R)) \geq |A(R)^*| - (2^n - 1) = |A(R)^*| - 2^n + 1,$$

by Remark 3.1.

Now, we show that

$$dim_M(\Omega(R)) \leq |Z(R)^*| - 2^n + 1.$$

We put

$A = \{(I_1, \ldots, I_n) \in V(\Omega(R)) \mid I_i \in \{0, R_1, \ldots, R_n\}$ for every $1 \leq i \leq n\} \cup$
$\{(Nil(R_1), \ldots, Nil(R_n))\}$

$W = A(R)^* - A$.

We show that W is a resolving set and consequently the metric basis for the graph $\Omega(R)$. For this, let $I, J \in A$ and $I \neq J$. Let $I = (I_1, \ldots, I_n)$ and $J = (J_1, \ldots, J_n)$. We show that $D(I|W) \neq D(J|W)$.

We have the following cases:

Case 1. $I = (Nil(R_1), \ldots, Nil(R_n))$ or $J = (Nil(R_1), \ldots, Nil(R_n))$. Without loss of generality, we may assume that $I \neq (Nil(R_1), \ldots, Nil(R_n))$ and $J = (Nil(R_1), \ldots, Nil(R_n))$. By Part (2) of Lemma 3.1, we have $D(J|W) = (1, 1, \ldots, 1)$.To continue the proof let $I' = ann(I)$. Since $I + I' = R$, I is not adjacent to I'. Since $|[I']| > 1$ (note that for every $X \in V(\Omega(R))$, $|[X]| > 1$), $W \cap [I'] \neq \emptyset$. This implies that for every $K \in W \cap [I']$, K is not adjacent to I and hence $D(I|W) \neq (1, 1, \ldots, 1)$. Therefore, $D(I|W) \neq D(J|W)$.

Case 2. $I \neq (Nil(R_1), \ldots, Nil(R_n))$ and $J \neq (Nil(R_1), \ldots, Nil(R_n))$.

Since $I \nsim J$, for some $1 \leq i \leq n$, $I_i = 0$ and $J_i = R_i$ or $J_i = 0$ and $I_i = R_i$. Without loss of generality, we may assume that $I_1 = 0$ and $J_1 = R_1$. So $I = (0, I_2, \ldots, I_n)$ and $J = (R_1, J_2, \ldots, J_n)$. Let $0 \neq K \subseteq Nil(R_1)$ and $X = (K, R_2, \ldots, R_n)$. This, clearly follows that, $X \in W$, $d(I, X) = 1$ and $d(J, X) = 2$. Therefore, $D(I|W) \neq D(J|W)$.

Since $|A| = 2^n - 1$, we have $|W| = |A(R)^*| - (2^n - 1) = |A(R)^*| - 2^n + 1$. So

$$dim_M(\Omega(R)) \leq |A(R)^*| - 2^n + 1.$$

□

Corollary 3.1. *Suppose that $R \cong R_1 \times \cdots \times R_n$, where R_i is an Artinian local ring such that for every $1 \leq i \leq n$, $|A(R_i)^*| = 1$. Then $dim_M(\Omega(R)) = 3^n - 2^n - 1$.*

Lemma 3.2. *Let $R \cong R_1 \times \cdots \times R_n \times F_1 \times \cdots \times F_m$, $n \geq 1$, $m \geq 1$ where each $(R_i, \mathfrak{m}_i)$ is an Artinian local ring with $\mathfrak{m}_i \neq (0)$ and each F_i is a field and let $S \cong F'_1 \times \cdots \times F'_{n+m}$ where each F'_i is a field. Then $dim_M(\Omega(R)[A]) = dim_M(\Omega(S)) = m + n$, where $A = \{(I_1, \ldots, I_{n+m}) \in V(\Omega(R)) \mid I_i \in \{0, R_1, \ldots, R_n, F_1, \ldots, F_m\}\} \cup \{(Nil(R_1), \ldots, Nil(R_n), 0, \ldots, 0)\}$.*

Proof. It is not hard to see that $\Omega(R)[A] \cong \Omega(S)$ and hence by Theorem 2.1, $dim_M(\Omega(R)[A]) = dim_M(\Omega(S)) = m + n$. □

Theorem 3.2. *Let $R \cong R_1 \times \cdots \times R_n \times F_1 \times \cdots \times F_m$, be a finite ring, $n \geq 1$, $m \geq 1$ where each $(R_i, \mathfrak{m}_i)$ is an Artinian local ring with $\mathfrak{m}_i \neq (0)$ and each F_i is a field. Then $dim_M(\Omega(R)) = |A(R)^*| - 2^{n+m} + m + 1$.*

Proof. Assume that $I = (I_1, \ldots, I_n)$ and $J = (J_1, \ldots, J_n)$ are vertices of $\Omega(R)$. Define the relation $\sim$ on $V(\Omega(R))$ as follows: $I \sim J$, whenever for every $1 \leq i \leq n$, "$I_i \subseteq Nil(R_i)$ if and only if $J_i \subseteq Nil(R_i)$".

Clearly, $\sim$ is an equivalence relation on $V(\Omega(R))$. The equivalence class of I is denoted by $[I]$. Suppose that X and Y are two elements of the equivalence class of I. Since $X \sim Y$, by part (2) of Lemma 3.1, we can easily get $N(X) = N(Y)$. Now, since the number of equivalence classes is $2^{n+m} - 1$, then

$$dim_M(\Omega(R)) \geq |A(R)^*| - (2^{n+m} - 1) = |A(R)^*| - 2^{n+m} + 1,$$

by Remark 3.1. Now, we put

$A = \{(I_1, \ldots, I_{n+m}) \in V(\Omega(R)) \mid I_i \in \{0, R_1, \ldots, R_n, F_1, \ldots, F_m\}\} \cup \{(Nil(R_1), \ldots, Nil(R_n), 0, \ldots, 0)\}$.

In fact, $|A| = 2^{n+m} - 1$ and for every equivalence class say $[I]$, $|[I] \cap A| = 1$. This means that if W is a resolving set for $\Omega(R)$, then $A(R)^* - A \subseteq W$.

Also, for every $n + 1 \leq i \leq n + m$, let

$$K_i = (R_1, R_2, \ldots, R_n, J_{n+1}, J_{n+2}, \ldots, J_{n+m})$$

such that $J_j = 0$ if $i = j$, and if $i \neq j$, $J_j = F_j$ and let

$B = \{K_{n+1}, K_{n+2}, \ldots, K_{n+m}\}$
$C = \{(0, 0, \ldots, 0, I_{n+1}, \ldots, I_{n+m}) \in V(\Omega(R)) \mid I_{n+j} \in \{0, F_1, \ldots, F_m\}$ for every $1 \leq j \leq m\} \cup \{(Nil(R_1), \ldots, Nil(R_n), 0, \ldots, 0)\}$.

Now, let $I = (I_1, \ldots, I_{n+m}) \in A(R)^* - A \subseteq W$. Then since $|[I]| > 1$, for some $1 \leq i \leq n$, $I_i \subseteq Nil(R_i)$. So by Lemma 3.1, every element of $A(R)^* - A$ is adjacent to all elements of $C \subseteq A$. Thus we need to add some elements of A to $A(R)^* - A$ to get W. For this by proof of Theorem 2.1 for reduced rings and Lemma 3.2, the best candidate is the set of elements of B. So $B \subseteq W$. This implies that

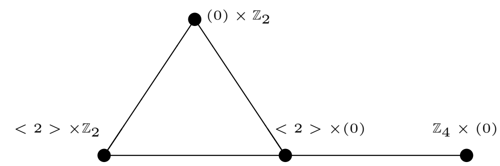
$$dim_M(\Omega(R)) \geq |A(R)^*| - 2^{n+m} + m + 1.$$

We prove that $W = \{A(R)^* - A\} \cup B$ is a resolving set and consequently the metric basis for the graph $\Omega(R)$. But this is clear by Theorem 2.1 and Lemma 3.2. $\square$

We close this section with the following example which is related to Theorem 3.2.

Example 3.1.

(1) Let $R = \mathbb{Z}_4 \times \mathbb{Z}_2$. Then $n = m = 1$ (in Theorem 3.2). We have $|A(R)^*| = 4$. By Theorem 3.2, $dim_M(\Omega(R)) = |A(R)^*| - 2^{n+m} + m + 1 = 4 - 2^2 + 1 + 1 = 2$. Also, by the following figure, we regain that $dim_M(\Omega(R)) = 2$.



$\Omega(\mathbb{Z}_4 \times \mathbb{Z}_2)$

(2) Let $R = \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. Then $n = m = 2$ (in Theorem 3.2). We have $|A(R)^*| = 34$.

$A = \{(0, \mathbb{Z}_4, \mathbb{Z}_2, \mathbb{Z}_2), (\mathbb{Z}_4, 0, \mathbb{Z}_2, \mathbb{Z}_2), (\mathbb{Z}_4, \mathbb{Z}_4, 0, \mathbb{Z}_2), (\mathbb{Z}_4, \mathbb{Z}_4, \mathbb{Z}_2, 0), (0, 0, \mathbb{Z}_2, \mathbb{Z}_2),$
$(0, \mathbb{Z}_4, 0, \mathbb{Z}_2), (0, \mathbb{Z}_4, \mathbb{Z}_2, 0), (\mathbb{Z}_4, 0, \mathbb{Z}_2, 0), (\mathbb{Z}_4, 0, 0, \mathbb{Z}_2), (\mathbb{Z}_4, \mathbb{Z}_4, 0, 0), (\mathbb{Z}_4, 0, 0, 0),$
$(0, \mathbb{Z}_4, 0, 0), (0, 0, \mathbb{Z}_2, 0), (0, 0, 0, \mathbb{Z}_2), ((2), (2), 0, 0)\ \}$.

Since $A(R)^* = \cup_{I \in A}[I]$ and for every $X, Y \in [I]$, $N(X) = N(Y)$, we must have $A(R)^* - A \subseteq W$ (in fact $X \in W$ or $Y \in W$). Now, let

$B = \{(\mathbb{Z}_4, \mathbb{Z}_4, \mathbb{Z}_2, 0), (\mathbb{Z}_4, \mathbb{Z}_4, 0, \mathbb{Z}_2)\}$,

$C = \{(0, 0, 0, \mathbb{Z}_2), (0, 0, \mathbb{Z}_2, 0), (0, 0, \mathbb{Z}_2, \mathbb{Z}_2), ((2), (2), 0, 0)\}$.

So by Lemma 3.1, every element of $A(R)^* - A$ is adjacent to all elements of C and since $|C| = 4$, we need to add at least two elements of A to $A(R)^* - A$ to get W. So we can let $B \subseteq W$. Therefore,

$W = \{((2), \mathbb{Z}_4, \mathbb{Z}_2, \mathbb{Z}_2), (\mathbb{Z}_4, (2), \mathbb{Z}_2, \mathbb{Z}_2), ((2), (2), \mathbb{Z}_2, \mathbb{Z}_2), ((2), (2), 0, \mathbb{Z}_2),$
$((2), (2), \mathbb{Z}_2, 0), ((2), \mathbb{Z}_4, 0, \mathbb{Z}_2), ((2), \mathbb{Z}_4, \mathbb{Z}_2, 0), (\mathbb{Z}_4, (2), \mathbb{Z}_2, 0), (\mathbb{Z}_4, (2), 0, \mathbb{Z}_2),$
$(\mathbb{Z}_4, \mathbb{Z}_4, \mathbb{Z}_2, 0), (\mathbb{Z}_4, \mathbb{Z}_4, 0, \mathbb{Z}_2), (\mathbb{Z}_4, (2), 0, 0), ((2), \mathbb{Z}_4, 0, 0), ((2), (0), \mathbb{Z}_2, \mathbb{Z}_2),$
$((2), 0, 0, \mathbb{Z}_2), ((2), 0, \mathbb{Z}_2, 0), (0, (2), \mathbb{Z}_2, \mathbb{Z}_2), (0, (2), 0, \mathbb{Z}_2), (0, (2), \mathbb{Z}_2, 0),$
$((2), 0, 0, 0), (0, (2), 0, 0)\}$.

We have:

$D((0, \mathbb{Z}_4, \mathbb{Z}_2, \mathbb{Z}_2)|W) = (1, 2, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((\mathbb{Z}_4, 0, \mathbb{Z}_2, \mathbb{Z}_2)|W) = (2, 1, 1, 1, 1, 2, 2, 1, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((0, 0, \mathbb{Z}_2, \mathbb{Z}_2)|W) = (1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((0, \mathbb{Z}_4, 0, \mathbb{Z}_2)|W) = (1, 2, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((0, \mathbb{Z}_4, \mathbb{Z}_2, 0)|W) = (1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((\mathbb{Z}_4, 0, \mathbb{Z}_2, 0)|W) = (2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((\mathbb{Z}_4, 0, 0, \mathbb{Z}_2)|W) = (2, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((\mathbb{Z}_4, \mathbb{Z}_4, 0, 0)|W) = (2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1)$,
$D((\mathbb{Z}_4, 0, 0, 0)|W) = (2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((0, \mathbb{Z}_4, 0, 0)|W) = (1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((0, 0, \mathbb{Z}_2, 0)|W) = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,
$D((0, 0, 0, \mathbb{Z}_2)|W) = (1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,

$D(((2),(2),0,0)|W) = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)$.

It is seen that for every $I, J \in A(R)^* - W$ with $I \neq J$, $D(I|W) \neq D(J|W)$ and so W is a resolving set and consequently the metric basis for the graph $\Omega(\mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$.

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